

Exam ALTAM

Date: Tuesday, October 24, 2023

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 6 questions numbered 1 through 6 with a total of 60 points.

The points for each question are indicated at the beginning of the question.
2. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.
3. Each question part or subpart should be answered in the provided Yellow Answer Booklet. Graders will only look at the work in the Yellow Answer Booklet.
4. The Excel file will not be uploaded for grading, and therefore will NOT BE GRADED. It should be used for looking up values for statistical functions and may be used for calculations.
5. If you use Excel for calculations, you should include as much information in the Yellow Answer Booklet as if you had used a calculator, including formulas and intermediate calculations where relevant. Written answers without sufficient support will not receive full credit.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas in the Yellow Answer Booklet. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, hand in all your written answer sheets to the Prometric Center staff. Be sure to hand in all your answer sheets because they cannot be accepted later.

BEGINNING OF EXAMINATION
****ADVANCED LONG-TERM ACTUARIAL MATHEMATICS****

1.

(12 points) XYZ Insurance issues fully discrete joint life 20-year endowment insurance policies to couples, each age 65, with independent future lifetimes. The sum insured is 100,000, payable at the end of the year of the first death, or at time 20 if both individuals survive to that time.

Premiums are payable annually while both lives survive, for a maximum of 20 years.

You are given the following assumptions for premiums and reserves:

- (i) Acquisition expenses are 500 plus 20% of the first year's premium.
 - (ii) Renewal expenses are 5% of each subsequent premium.
 - (iii) Mortality follows the Standard Ultimate Mortality Model.
 - (iv) Gross premiums are calculated using the equivalence principle.
 - (v) $i = 0.05$
 - (vi) Reserves are gross premium policy values.
- (a) (1 point) Calculate ${}_{20}E_{65:65}$.
- (b) (2 points) Show that the gross premium for the policy is 4,750 to the nearest 10. You should calculate the value to the nearest 1.
- (c) (1 point) Show that the gross premium policy value at time 10 is 38,250 to the nearest 10. You should calculate the value to nearest 1.

At time 10, XYZ Insurance had 100 identical and independent policies in force. During the 11th policy year, the actual experience was as follows:

- (i) The interest rate earned was 5.5%.
- (ii) Renewal expenses were 5% of the premium.
- (iii) There were 4 claims.
- (iv) Each claim had a termination expense of 100.

1. Continued

You are given that the gross premium policy value at time 11 is 42,811.92.

- (d) (2 points) Calculate the total gain in the 11th policy year for the portfolio.
- (e) (4 points) Calculate the gain by source in the 11th policy year, in the order of (1) Interest, (2) Mortality, and (3) Expenses.

At time 19, there were 40 policies remaining in force. During the 20th policy year, a pandemic occurred, such that the actual experience was:

- The interest rate earned was 5.0%.
- Renewal expenses were 5% of the premium.
- There were 35 claims.
- There were no termination expenses.

- (f) (2 points)
- (i) Show that the total gain in the 20th policy year for the portfolio is zero.
- (ii) Explain why there is no mortality loss despite the pandemic.

2.

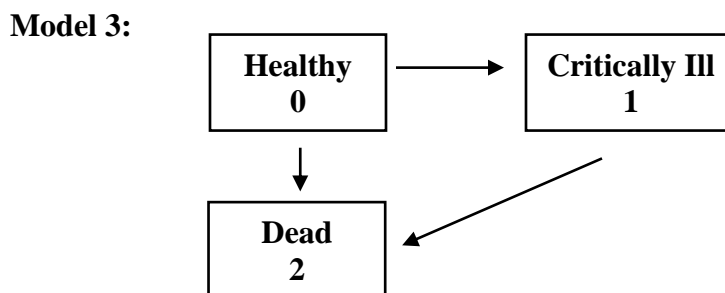
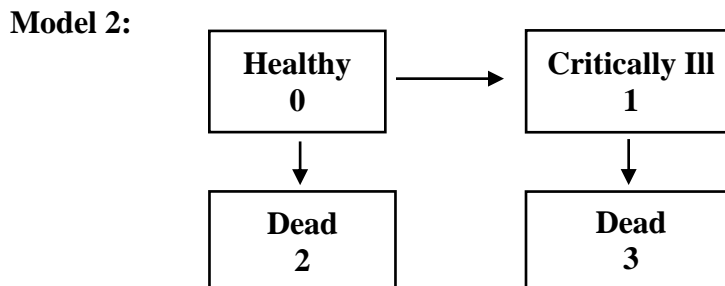
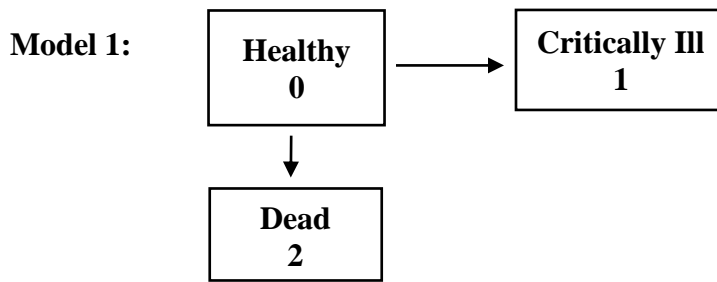
(12 points) An insurance company offers the following range of options for critical illness (CI) insurance riders attached to a life insurance policy. In each case the critical illness benefit would be paid immediately upon diagnosis of a covered critical illness.

Option A: Partially accelerated death benefit

Option B: Fully accelerated death benefit

Option C: Additional lump sum benefit on CI diagnosis

(a) (2 points) For each of the options above, state with reasons which of the following models could be used to calculate premiums and reserves.



2. Continued

A special 10-year term insurance policy issued to a Healthy life age 50 pays 100,000 immediately upon death of the policyholder, and an additional lump sum benefit of 50,000 immediately upon diagnosis of critical illness. Premiums are payable annually while the policyholder is Healthy. You are given that the insurer uses Model 3 for analyzing this policy.

- (b) (1 point)
- (i) Write down an integral expression for the probability that a new policyholder dies before the policy expires.
 - (ii) Write down an integral expression for the expected present value of the death benefit.

The insurer values the contract payments using force of interest of $\delta = 0.05$. You are given the following table of actuarial functions under Model 3, at $\delta = 0.05$.

x	${}_{10}P_x^{00}$	${}_{10}P_x^{01}$	$\mu_x^{0\bullet}$	\bar{a}_x^{00}	\bar{A}_x^{01}	\bar{A}_x^{02}	\bar{A}_x^{12}
50	0.90880	0.04152	0.00562	13.1491	0.21846	0.21878	0.30227
60	0.75019	0.13565	0.01542	10.0781	0.33692	0.33714	0.41861

- (c) (2 points) Show that $\ddot{a}_{50:\overline{10}|}^{00} = 7.8$ to the nearest 0.1, using Woolhouse's 3-term formula. You should calculate the value to the nearest 0.001.
- (d) (3 points) Show that the annual net premium is 495 to the nearest 5. You should calculate the value to the nearest 0.1.
- (e) (2 points) You are given the following additional information for a policy in force at time 5:

t	${}_tV^{(0)}$	${}_tV^{(1)}$	μ_{50+t}^{01}	μ_{50+t}^{02}
5	702.6	5322.0	0.004265	0.004835
6	714.3	4562.4	0.004845	0.005234

Calculate $\frac{d}{dt} {}_tV^{(0)}$ at time 5, immediately after the premium payment.

- (f) (2 points) Sketch the graph of ${}_tV^{(0)}$ as a function of t for $5 \leq t \leq 6$. You should mark key numerical values on each axis.

3.

(9 points) An insurer sells a 3-year single premium Variable Annuity contract, with the following provisions:

- (i) The policyholder deposits the 10,000 premium at the start of the contract.
- (ii) The insurer deducts a 2% front-end charge from the premium before allocating the remainder to the policyholder fund.
- (iii) At the start of each year after the first, the insurer deducts a management charge of 1% of the fund value.
- (iv) The death benefit under the contract is 110% of the fund value at the end of the year of death, payable at that time.
- (v) If the policyholder surrenders during the first year, they receive 85% of their fund value at the end of the year. In subsequent years, the policyholder receives the full fund value at the end of the year of surrender.
- (vi) The contract has a Guaranteed Minimum Maturity Benefit (GMMB) rider, which is 100% of the premium.

An actuary conducts a stress test of this contract using the following assumptions:

- Initial expenses are 200 at contract inception.
- Renewal expenses are 0.6% of the policyholder's fund value (before management charge deductions) incurred at the start of the second and third policy years.
- Mortality rates are 0.05 each year.
- At the end of the first and second years, 10% of the surviving policyholders surrender their policies.
- The policyholder's fund earns 0% in the first year, 0% in the second year, and 5% in the third year.
- The insurer's funds earn 3% in each year.
- The insurer holds a reserve of 300, 200, and 100 at the start of years 1, 2, and 3 respectively, for each contract in force at the start of the year.

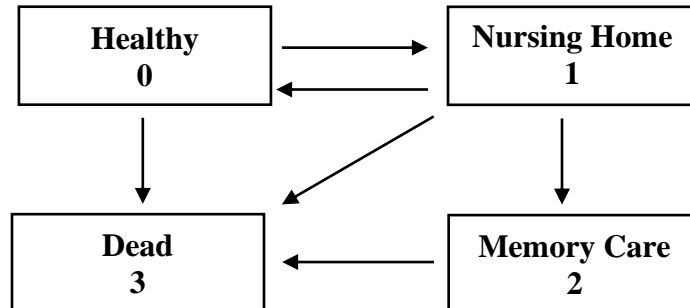
3. Continued

- (a) (1 point) Show that the policyholder fund value at the end of year 3, assuming the policy remains in force, is 10,100 to the nearest 100. You should calculate the value to the nearest 1.
- (b) (5 points)
- (i) (3.5 points) Determine the profit vector of the contract using the stress test assumptions.
- (ii) (1.5 points) Calculate the net present value of the profit per policy issued, using a hurdle rate of 10% per year.
- (c) (3 points) The actuary is concerned about the GMMB risk.
- (i) One colleague suggests selling more policies to mitigate the risk. Explain why this strategy would be ineffective.
- (ii) Another colleague suggests a hedging strategy for the GMMB. The price at issue of the required put option is 900.

Describe one advantage and one disadvantage of hedging the guarantee, compared with holding reserves.

4.

(9 points) Twenty individuals with independent future lifetimes, all exact age 70, are subject to the following multi-state nursing home model:



You are using maximum likelihood estimation to estimate model transition intensities, assuming constant transition intensities.

You are given:

- (i) Life A is in State 0 at age 70 and remains in State 0 until she dies at age 70.9.
 - (ii) Life B is in State 1 at age 70. At age 70.2, she moves into State 0 where she remains until age 70.5. At age 70.5, she moves into State 1 and remains there until she dies at age 70.9.
 - (iii) Life C is in State 0 at age 70. At age 70.3, she moves into State 1 where she remains until age 70.6. At age 70.6, she moves into State 2 and remains there until age 71.
 - (iv) Life D is in State 0 at age 70. At age 70.5, she moves into State 1 where she remains until age 70.7 at which point she dies.
 - (v) Six additional lives are in State 0 at age 70 and remain there until age 71.
 - (vi) Nine additional lives are in State 1 at age 70 and remain there until age 71.
 - (vii) One additional life is in State 2 at age 70 and remains there until age 71.
- (a) (1 point) Explain in words why the contribution to the likelihood of a life who remains in State 0 from age 70 to age 71 is $e^{-(1)(\mu_{70}^{01} + \mu_{70}^{03})}$.

4. Continued

- (b) (2 points) Write an expression for the contribution to the likelihood of Life B expressed entirely in terms of transition intensities.
- (c) (2 points) Calculate the maximum likelihood estimate of μ_{70}^{01} .
- (d) (2 points) Calculate the maximum likelihood estimate of ${}_1p_{70}^{\overline{00}}$.
- (e) (1 point) Calculate the estimated standard deviation of the maximum likelihood estimate of μ_{70}^{01} .
- (f) (1 point) The maximum likelihood estimate of μ_{70}^{23} is 0.

Explain why this transition intensity is estimated to be 0 despite the transition from State 2 to State 3 being a possible transition in the model.

5.

(8 points) An insurer sells both Type A and Type B universal life insurance policies to lives age 75.

You are given the following information about these policies:

- (i) The cost of insurance rates for the policy are 100% of the mortality rates in the Standard Ultimate Mortality Model.
 - (ii) The interest rate used to discount the cost of insurance charge is 4%.
 - (iii) The expense charges at the beginning of each policy year are 100 plus 10% of premium.
 - (iv) The credited interest rate during all policy years is 7%.
 - (v) The corridor factor in all years is 1.05.
- (a) (1 point) Explain why Type A universal life policies are more commonly affected by corridor factors than Type B universal life policies.

Consider a Type B universal life policy, with a death benefit of 100,000 plus the account value. The premium paid at the beginning of the first policy year is 10,000.

- (b) (1 point) Show that the cost of insurance charge for the first policy year is 1800 to the nearest 100. You should calculate the value to the nearest 1.
- (c) (1 point) Calculate the account value of the policy at the end of the first policy year.

Now consider a Type A universal life policy with a face amount of 100,000. The premium paid at the beginning of the first policy year is 10,000.

- (d) (2 points) Calculate the account value of this policy at the end of the first policy year.
- (e) (3 points) The account value at the end of the 8th policy year for the Type A policy is 81,618.64. The policyholder pays a 10,000 premium at the beginning of the 9th policy year. Calculate the account value at the end of the 9th policy year.

6.

(10 points) You are the actuary for a defined benefit pension plan that offers a Final Average Salary (FAS) pension. The final average salary is the average of the earnings in the final two years, and the pension is paid as a whole life annuity due, payable monthly from retirement. The accrual rate is 2%.

You are given the following valuation assumptions:

- (i) Decrements are assumed to follow the Standard Service Table up to age 61. All lives in force at age 61 retire immediately.
- (ii) All retirements not occurring at exact ages 60 or 61 are assumed to occur at age 60.5.
- (iii) $i = 0.05$
- (iv) Under the post-retirement mortality assumptions:
 $\ddot{a}_{60}^{(12)} = 14.441$ $\ddot{a}_{60.5}^{(12)} = 14.315$ $\ddot{a}_{61}^{(12)} = 14.186$
- (v) The valuation date is 12/31/2022.
- (vi) There were no salary increases in 2021 or 2022.
- (vii) Salaries increased by 10% on 1/1/2023.
- (viii) The valuation uses the Traditional Unit Credit funding method.

The plan has two members. Their information at the valuation date is given in the following table:

Employee	Age	Number of years of service	FAS at valuation
A	35	10	60,000
B	60	30	100,000

- (a) (4 points)
 - (i) Show that the Actuarial Liability for Employee A's retirement benefits is 21,000 to the nearest 1,000. You should calculate the value to the nearest 1.
 - (ii) You are given that Employee B did not retire at exact age 60. Show that the Actuarial Liability for Employee B's retirement benefits is 810,000 to the nearest 1,000. You should calculate the value to the nearest 1.

6. Continued

(b) (4 points)

- (i) (1 point) Calculate the Normal Contribution for Employee A's retirement benefits.
- (ii) (2.5 points) Calculate the Normal Contribution for Employee B's retirement benefits.
- (iii) (0.5 points) The Aggregate Normal Contribution Rate is the total Normal Contribution divided by the total annual salary rate as of 1/1/2023.

Calculate the Aggregate Normal Contribution Rate for the plan.

(c) (2 points) A new entrant who is age 35 joins the plan on 12/31/2022.

- (i) State with reasons whether the plan's total Actuarial Liability for retirement benefits would increase, decrease, or stay the same.
- (ii) State with reasons whether the Aggregate Normal Contribution Rate for retirement benefits would increase, decrease, or stay the same.

****END OF EXAMINATION****